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Non-extensive resonant reaction rates in astrophysical plasmas

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Abstract. We study two different physical scenarios of thermonuclear reactions in stellar plasmas proceeding through a narrow resonance at low energy or through the low-energy wing of a wide resonance at high energy. Correspondingly, we derive two approximate analytical formulae in order to calculate thermonuclear resonant reaction rates inside very coupled and non-ideal astrophysical plasmas in which non-extensive effects are likely to arise. Our results are presented as simple first-order corrective factors that generalize the well-known classical rates obtained in the framework of Maxwell-Boltzmann statistical mechanics. As a possible application of our results, we calculate the dependence of the total corrective factor with respect to the energy at which the resonance is located, in an extremely dense and non-ideal carbon plasma.

PACS. 24.30.-v Resonance reactions – 26.50.+x Nuclear physics aspects of novae, supernovae, and other explosive environments – 05.90.+m Other topics in statistical physics, thermodynamics, and nonlinear dynamical systems

1 Introduction

Cussons, Langanke and Liolios [1] proposed, on the basis of experimental measurements at energy $E \sim 2.4\,\mathrm{MeV}$, that the resonant behavior of the stellar $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ fusion cross-section could continue down to the astrophysical energy range, thus leading rise, beside the usual Debye-Hückel screening [2] (whose corrective factor is f_{S}), to a further plasma resonant screening effect (conventionally described by a suitable f_{RS} factor). While $f_{\mathrm{S}} > 1$ enhances the reaction rate, it has been pointed out that $f_{\mathrm{RS}} < 1$, *i.e.* the resonant screening effect is likely to reduce the rate. The reduction of the resonant rate due to a resonant screening correction amounts to 11 orders of magnitude at the resonant energy of 400 keV, influencing the carbon ignition density in white dwarfs. Itoh *et al.* [3] have shown that, using an effective screening potential obtained by one-component–plasma (OCP) Monte Carlo experiments, the overall effect does indeed strongly enhance the carbon-carbon reaction rate by a considerable amount (*i.e.* $f_{\mathrm{T}} \equiv f_{\mathrm{S}} \cdot f_{\mathrm{RS}} \gg 1$) because of the global screening domination that amounts to an enhancement of the rate by 12 orders of magnitude, with important implications for hydrostatic burning in carbon white dwarfs. Given the $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ reaction, the current hypothesis [1] is that the entrance channel width is much smaller than the total resonance width, the latter being much smaller than the resonance energy. The same picture could possibly apply to other fusion reactions between medium-weighted nuclei (*e.g.*, to the $^{16}\mathrm{O} + {}^{16}\mathrm{O}$ reaction, which could be an active burning stage in some white dwarfs [4]).

The previous discussion refers to extremely dense stellar plasmas, characterized by a temperature $T \sim 10^8\,\mathrm{K}$, a mass density $\rho \sim 10^9\,\mathrm{g \cdot cm^{-3}}$ and a plasma parameter $\Gamma < 178$. In these physical conditions, we expect that non-extensive effects could also arise. We briefly recall that such plasmas show deviations from the several assumptions that are the basis of Maxwell-Boltzmann distribution. Long-range many-body nuclear correlations and memory effects, among others, can be sufficient to justify the use of a distribution function which slightly deviates from the standard Maxwell-Boltzmann one [5] (see also, for example, ref. [6] for a discussion on physical conditions in which non-extensivity needs to be taken into account). Our aim is to derive a simple first-order formula in order to express non-extensive corrections for reactions proceeding through narrow resonances: we will consider this case in subsect. 3.1.

In addition to the above resonant reactions in white dwarfs, let us mention few resonant reactions occurring in the stellar interior (like the Sun interior), where no carbon burning is active. It is well known that, along with the reactions of the proton-proton chain, many reactions

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of the CNO cycle are presently believed to be non-resonant at the relevant stellar energies around the maximum E_0 of the Gamow peak (whose order of magnitude is $E_0 \sim 10\,\mathrm{keV}$ inside the Sun's core). However, the $^{17}\mathrm{O}(p,\alpha)^{14}\mathrm{N}$ reaction, which belongs to the CNO-II subcycle, shows a very narrow resonance ($^{18}\mathrm{F}$ being the compound nucleus) located at an energy between 65 keV and 75 keV, with a proton partial width $\Gamma_p \simeq 22\,\mathrm{neV}$; this might be taken into account when solar-model calculations are carried out (see [7] and references therein). Moreover, one could actually point out that many non-resonant reactions in the CNO cycle proceed indeed through the low-energy wing of a wide resonance located at energy $E_\mathrm{R} \gg E_0$, as was already observed by Clayton [8] (among others, we recall also the $^{14}\mathrm{N}(p,\gamma)^{15}\mathrm{O}$ reaction; its S factor has been very recently measured by the LUNA group [9]). In this case, we could adopt a very reliable resonant formalism that allows us to evaluate the astrophysical factor of the reaction at the Gamow peak energy, $S(E_0)$, by relying exclusively on the experimental data of the resonance at the relatively high energy E_R.

For the sake of completing this discussion, let us consider the $^{12}\mathrm{C}(p,\gamma)^{13}\mathrm{N}$ reaction (CNO-I subcycle). At its Gamow energy in the Sun, $E_0 = 24.68\,\mathrm{keV}$, this reaction seems to be non-resonant [10], but it shows a wide resonance at energy $E_\mathrm{R} = 424\,\mathrm{keV} \gg E_0$, with a total width $\Gamma_\mathrm{T} \simeq 40\,\mathrm{keV}$, and an electromagnetic channel width $\Gamma_\gamma \simeq 0.77\,\mathrm{eV}$. It is an easy matter to express the astrophysical factor $S(E_0)$ by means of E_R, Γ_T and Γ_γ only, through a Breit-Wigner approximation formula [11].

As has been pointed out (see, for example, refs. [12, 13]), a very slight non-extensive deformation could also arise in the Sun interior: we will derive a first-order non-extensive corrective factor for nuclear reactions proceeding through the low-energy wing of a wide resonance in subsect. 3.2.

From the previous discussion it follows that it is useful to develop an analytical theory dealing with reactions that proceed through narrow or wide resonances; this work was already achieved in ref. [8]. In this paper we extend the known classical results to the case in which non-extensive corrections arise, and we present a possible application to the $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ reaction in a white dwarf's plasma.

2 Classical resonant reaction rates

In this section, we briefly review some of the known results about the analytical reaction rate calculations in astrophysical plasmas, that were obtained in the framework of the classical Maxwell-Boltzmann statistics (MB) [8,10].

Let us consider a thermonuclear reaction between two nuclei i and j. We define the classical reaction rate, r_{ij}^{MB}, by

$$r_{ij}^{\mathrm{MB}} = \frac{N_i N_j}{1+\delta_{ij}} \langle v_{ij}\sigma_{ij}\rangle_{\mathrm{MB}} = \frac{N_i N_j}{1+\delta_{ij}} \int_0^{+\infty} \phi_{\mathrm{MB}}(E) v_{ij}(E) \sigma_{ij}(E) \mathrm{d}E, \quad (1)$$

where N_i, N_j are the particle densities, E is the relative energy (in the center-of-mass frame of reference) at which the reaction occurs, v_{ij} is the relative velocity between two fusing nuclei, σ_{ij} is the reaction cross-section, ϕ_{MB} is the Maxwell-Boltzmann energy distribution function, and $\delta_{ij} = 1$ if nuclides i and j are identical, $\delta_{ij} = 0$ otherwise.

As far as resonant reactions are concerned, we can distinguish two different cases of physical interest. Let E_R be the resonance energy, E_0 the Gamow energy and Γ_T the total resonance width. If the following conditions are satisfied,

$$\begin{cases} E_\mathrm{R} \approx E_0, \\ \Gamma_\mathrm{T} \ll E_\mathrm{R}, \end{cases} \quad (2)$$

the reaction proceeds through a narrow resonance at low energy (this case will be labelled as "r"). The $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ reaction occurring inside a white dwarf, beside the hypotheses already discussed in sect. 1, belongs to this first scenario.

On the contrary, if the following inequalities are satisfied,

$$\begin{cases} E_\mathrm{R} > E_0 \text{ or } E_\mathrm{R} \gg E_0, \\ \Gamma_\mathrm{T} > E_0, \end{cases} \quad (3)$$

the resonance is said to be a wide resonance (this case will be labelled as "R"). The $^{12}\mathrm{C}(p,\gamma)^{13}\mathrm{N}$ reaction inside the Sun belongs to this second scenario.

2.1 Narrow resonances at low energy (MB, r)

If the conditions in eq. (2) hold, the approximate reaction rate reads [8]

$$r_{ij}^{\mathrm{MB,r}} = (2\pi)^{3/2} \frac{N_i N_j}{1+\delta_{ij}} \frac{\omega_{ij}\hbar^2}{(\mu_{ij} \mathrm{k_B} T)^{3/2}} \times \frac{\Gamma_\mathrm{in}(E_\mathrm{R})\Gamma_\mathrm{out}}{\Gamma_\mathrm{T}} \exp\left(-\frac{E_\mathrm{R}}{\mathrm{k_B}T}\right), \quad (4)$$

where $\Gamma_\mathrm{in}(E_\mathrm{R})$ is the entrance channel width, Γ_out is the exit channel width, $\Gamma_\mathrm{T} \sim \Gamma_\mathrm{in} + \Gamma_\mathrm{out}$ is the total resonance width, $\mathrm{k_B}T$ is the plasma thermal energy and μ_{ij} is the reduced mass of the two-body system $i+j$. In eq. (4), the quantum factor ω_{ij} is defined by [14]

$$\omega_{ij} = \frac{2J_\mathrm{C}+1}{(2J_i+1)(2J_j+1)},$$

where J_C is the total quantum angular momentum of the compound nucleus and J_i, J_j are the spin numbers of the interacting nuclei i and j.

2.2 Wide resonances (MB, R)

If the physical conditions in eq. (3) hold, the approximate reaction rate now reads [8]

$$r_{ij}^{\mathrm{MB,R}} = \frac{2^{3/2}\pi}{3^{1/2}} \frac{N_i N_j}{1+\delta_{ij}} \frac{\omega_{ij}\hbar^2 E_0^{1/2}}{\mu_{ij}^{3/2} \mathrm{k_B}T} \times \frac{\Gamma_\mathrm{in}\Gamma_\mathrm{out}}{(E_0 - E_\mathrm{R})^2 + (\Gamma_\mathrm{T}/2)^2} \exp\left(-\frac{3E_0}{\mathrm{k_B}T}\right). \quad (5)$$

Equation (5) has been obtained by writing the astrophysical factor S_{ij} as

$$S_{ij}(E) = \frac{\pi}{2}\frac{\hbar^2}{\mu_{ij}}\frac{\omega_{ij}\Gamma_{\rm in}\Gamma_{\rm out}}{(E-E_{\rm R})^2+(\Gamma_{\rm T}/2)^2}\,, \qquad (6)$$

through a well-known Breit-Wigner approximation formula, which is supposed to hold if the energy E is close to the resonance energy $E_{\rm R}$.

3 Non-extensive resonant reaction rates

In refs. [5,6,13] it has been shown that a coherent model describing a given stellar core might deal with slightly coupled plasmas, non-Markovian random walks, many-body collisions and memory effects. From this point of view, the classical theory founded on the Maxwell-Boltzmann statistical mechanics should be considered a first-order approximation (a very good one indeed inside the Sun's core). A further approximation, to which we are referring from now on, lies on the more general picture of non-extensive statistical mechanics [15,16].

Here we limit ourselves to briefly outline, by means of three different approaches, how non-standard statistics can arise. In the Fokker-Planck equation context, we can introduce corrections to the friction and diffusion coefficients, considering their expressions to the next order in the velocity variable; a stationary solution is the Tsallis non-extensive distribution. In a plasma, each particle is affected by the total electric field distribution of the other charges; the total microfields have a relatively small random component, generally show long-time correlations and generate anomalous diffusion. All these effects imply a deviation from the Maxwell-Boltzmann distribution whose entity depends on the plasma parameter and on an ion-ion correlation parameter. Correlations among the collective modes can lead to a long-time asymptotic behavior of the velocity correlations of the ions and to anomalous diffusion, that are related to generalized entropy and generate non-Maxwellian probability distributions.

In this new picture, the reaction rates are defined in the same way as in eq. (1), but now the non-extensive energy distribution function $\phi_{\rm NE}(E)$ is used, instead of the usual Maxwell-Boltzmann distribution $\phi_{\rm MB}(E)$.

It has been shown [6,12,13] that the analytical relationship linking $\phi_{\rm NE}$ and $\phi_{\rm MB}$ can be cast, to first order of approximation, in the following fashion:

$$\phi_{\rm NE}(E) = \left(1+\frac{15}{4}\delta\right)\phi_{\rm MB}(E)\exp\left[-\left(\frac{E}{{\rm k_B}T}\right)^2\delta\right], \quad (7)$$

provided that $|\delta| \ll {\rm k_B}T/E$ (or $|\delta| \ll {\rm k_B}T/E_0$, for computational purposes). The order of magnitude of the deformation parameter δ is 10^{-3}–10^{-2} inside the Sun, but it can be higher in extremely dense stellar plasmas (*e.g.*, in white dwarfs). The δ-parameter is linearly related to the q entropic parameter that appears in the non-extensive formalism [15]: the relationship between them two is $\delta = (1-q)/2$.

We can distinguish two physically relevant cases: if $\delta < 0$, the high-energy tail of the $\phi_{\rm NE}$ distribution function is increased with respect to $\phi_{\rm MB}$ (super-extensivity), while if $\delta > 0$, the high-energy tail is depleted (sub-extensivity).

Now we are able to develop the entire non-extensive resonant formalism of the thermonuclear reaction rates.

3.1 Narrow resonances at low energy (NE, r)

From eqs. (1) and (7), the non-extensive rate of a given reaction proceeding through a narrow resonance at low energy reads

$$r_{ij}^{\rm NE,r} = \frac{N_iN_j}{1+\delta_{ij}}\left(1+\frac{15}{4}\delta\right)\int_0^{+\infty}\phi_{\rm MB}(E) \times v_{ij}(E)\sigma_{ij}(E)\exp\left[-\left(\frac{E}{{\rm k_B}T}\right)^2\delta\right]{\rm d}E. \quad (8)$$

Starting from the hypotheses in eq. (2) we can state that, in the energy interval $E_{\rm R}-\Gamma_{\rm T} < E < E_{\rm R}+\Gamma_{\rm T}$, the integral in eq. (8) is strongly ruled by the reaction cross-section function $\sigma_{ij}(E)$ only. Thus we can write, with very good approximation, the following result:

$$r_{ij}^{\rm NE,r} = r_{ij}^{\rm MB,r}\left(1+\frac{15}{4}\delta\right)\exp\left[-\left(\frac{E_{\rm R}}{{\rm k_B}T}\right)^2\delta\right]. \quad (9)$$

If $|\delta| \ll ({\rm k_B}T/E_{\rm R})^2$, we can linearize eq. (9), and the first-order formula is

$$r_{ij}^{\rm NE,r} = r_{ij}^{\rm MB,r}[1+C_1({\rm k_B}T,E_{\rm R})\delta], \quad (10)$$

where

$$C_1({\rm k_B}T,E_{\rm R}) = \frac{15}{4}-\left(\frac{E_{\rm R}}{{\rm k_B}T}\right)^2.$$

Therefore, our final result in the case of narrow resonances at low energy is expressed by

$$r_{ij}^{\rm NE,r} = r_{ij}^{\rm MB,r}\left[1+\frac{15}{4}\delta-\left(\frac{E_{\rm R}}{{\rm k_B}T}\right)^2\delta\right]. \quad (11)$$

If the condition

$$\frac{E_{\rm R}}{{\rm k_B}T} \approx \frac{E_0}{{\rm k_B}T} > \sqrt{\frac{15}{4}} \simeq 1.936 \quad (12)$$

is satisfied, then $C_1({\rm k_B}T,E_{\rm R}) < 0$. In the Sun interior, the thermal energy is ${\rm k_B}T \simeq 1.36\,{\rm keV}$ and E_0 is in the energy interval $24\,{\rm keV} < E_0 < 29\,{\rm keV}$ for the CNO cycle, and $E_0 \approx 6\,{\rm keV}$ for the $p+p \to d+e^+ +\nu_e$ reaction. Thus, in the Sun's core, and in many other stellar plasmas of interest, eq. (12) is always satisfied; then, from eq. (10), we can actually state that the non-extensive rate is increased or diminished with respect to the classically calculated rate, whether $\delta < 0$ or $\delta > 0$ (as already mentioned in sect. 3).

3.2 Wide resonances (NE, R)

In the case of a generic reaction (no matter if resonant or not), the following formula linking $r_{ij}^{\rm NE}$ and $r_{ij}^{\rm MB}$ holds:

$$r_{ij}^{\rm NE}=r_{ij}^{\rm MB}\frac{S_{ij}(\tilde{E}_0)}{S_{ij}(E_0)}\left(1+\frac{15}{4}\delta-\frac{7}{3}\delta\frac{E_0}{{\rm k_B}T}\right)\exp(-\Delta_{ij}), \quad (13)$$

where

$$\Delta_{ij}\equiv\Delta_{ij}(\delta,\tilde{E}_0)=-\frac{3E_0}{{\rm k_B}T}\times\left[1-\left(1+\frac{5}{3}\delta\frac{\tilde{E}_0}{{\rm k_B}T}\right)\left(1+2\delta\frac{\tilde{E}_0}{{\rm k_B}T}\right)^{-2/3}\right], \quad (14)$$

and

$$\tilde{E}_0=E_0\left(1+2\delta\frac{\tilde{E}_0}{{\rm k_B}T}\right)^{-2/3}. \quad (15)$$

Equations (13) and (14) express the first-order correction to the classical reaction rate $r_{ij}^{\rm MB}$, due to the deformed distribution function that has been already defined in eq. (7). Besides, eq. (15) shows implicitly the relationship between the new (deformed) Gamow energy $\tilde{E}_0$, and the classical one E_0. A complete proof of these equations can be found in ref. [12], in which the authors started from an *ad hoc* assumption of the deformed distribution function, $\phi_{\rm NE}(E)$, regardless of any statistical basis. In this paper, we adopt the same results, but lying on the ground of non-extensive statistical mechanics (as was outlined, for example, in ref. [13]).

In eq. (13), the physical properties of the reaction are summarized in the ratio between the astrophysical factor at the new Gamow energy, $S_{ij}(\tilde{E}_0)$, and the same factor at E_0 energy, $S_{ij}(E_0)$. Our aim is to express that ratio as a function of $E_{\rm R}$ and $\Gamma_{\rm T}$ only. The deformation parameter δ is assumed to satisfy the following inequality:

$$|\delta|\ll\frac{{\rm k_B}T}{E_0}. \quad (16)$$

Then we can write, as a formal expansion in δ,

$$S_{ij}(\tilde{E}_0)\approx S_0+S_1\delta. \quad (17)$$

By writing eq. (17), we have substantially relied upon the physical properties already stated in eq. (3), that explicitly define the wide-resonance scenario. The conditions $E_0\ll E_R$ (and thus $\tilde{E}_0\ll E_{\rm R}$) and $\Gamma_{\rm T}>E_0$ allow us to expand the astrophysical factor $S_{ij}(E)$ as a formal series in δ, retaining first order only; in fact, in this case, its functional dependence on energy is moderate, at least in the low-energy wing of a broad resonance at high energy (S_{ij} is almost constant). This situation is very similar to the non-resonant formalism for which eq. (17) is a suitable approximation.

Before proceeding, we want to give some emphasis to the following point: eq. (17), together with subsequent calculations, cannot apply to narrow resonances for which conditions in eq. (2) hold, because in this case $S_{ij}(E)$ shows a strong dependence on energy, being a resonant function itself and, as a consequence, a linear theory lying on eq. (17) is not appropriate any more. It would be necessary a complete knowledge of $S_{ij}(E)$, in order to apply the formalism that we are developing here to a narrow resonance; that is the main reason why we adopted a different, but far simpler, treatment for this case, as was previously discussed in subsect. 3.1. Therefore the result we are obtaining in this subsection cannot produce, as a particular case, the result for the narrow resonance.

Now we can look for an analytical expression for the two coefficients S_0 and S_1 appearing in eq. (17). From eq. (6), we immediately obtain that

$$S_0=S_{ij}(E_0)=\frac{\pi}{2}\frac{\hbar^2}{\mu_{ij}}\frac{\omega_{ij}\Gamma_{\rm in}\Gamma_{\rm out}}{(E_0-E_{\rm R})^2+(\Gamma_{\rm T}/2)^2}. \quad (18)$$

On the contrary, the first-order coefficient in eq. (17) reads

$$S_1=-\pi\omega_{ij}\frac{\hbar^2}{\mu_{ij}}\frac{(E_0-E_{\rm R})\Gamma_{\rm in}\Gamma_{\rm out}}{[(E_0-E_{\rm R})^2+\Gamma_{\rm T}^2/4]^2}\left.\frac{{\rm d}\tilde{E}_0}{{\rm d}\delta}\right|_{\delta=0}. \quad (19)$$

In order to calculate the analytical expression for the ${\rm d}\tilde{E}_0/{\rm d}\delta|_{\delta=0}$ derivative, we differentiate both members of eq. (15) with respect to δ, obtaining

$$\frac{{\rm d}\tilde{E}_0}{{\rm d}\delta}=-\frac{4}{3}\frac{E_0}{{\rm k_B}T}\left(\tilde{E}_0+\frac{{\rm d}\tilde{E}_0}{{\rm d}\delta}\delta\right)\left(1+2\delta\frac{\tilde{E}_0}{{\rm k_B}T}\right)^{-5/3},$$

from which, without any approximation, it follows that

$$\left.\frac{{\rm d}\tilde{E}_0}{{\rm d}\delta}\right|_{\delta=0}=-\frac{4}{3}\frac{E_0^2}{{\rm k_B}T}. \quad (20)$$

Now we can rewrite eq. (17), using the results of eqs. (18) and (20), as

$$S_{ij}(\tilde{E}_0)\approx S_{ij}(E_0)\left(1+\frac{S_1}{S_0}\delta\right)$$

and then, the $S_{ij}(\tilde{E}_0)/S_{ij}(E_0)$ ratio eventually becomes

$$\frac{S_{ij}(\tilde{E}_0)}{S_{ij}(E_0)}=1+\frac{8}{3}\delta\frac{(E_0-E_{\rm R})E_0}{(E_0-E_{\rm R})^2+\Gamma_{\rm T}^2/4}\frac{E_0}{{\rm k_B}T}. \quad (21)$$

Placing the previous result of eq. (21) into eq. (13), it is clear that the non-extensive reaction rate, to first order, reads

$$r_{ij}^{\rm NE,R}=r_{ij}^{\rm MB,R}\exp(-\Delta_{ij})\left[1+\frac{15}{4}\delta-\frac{7}{3}\delta\frac{E_0}{{\rm k_B}T}+\frac{8}{3}\delta\frac{(E_0-E_{\rm R})E_0}{(E_0-E_{\rm R})^2+\Gamma_{\rm T}^2/4}\frac{E_0}{{\rm k_B}T}\right]. \quad (22)$$

A further approximation, under the hypothesis stated in eq. (16), is

$$\exp(-\Delta_{ij})\approx 1-\left(\frac{E_0}{{\rm k_B}T}\right)^2\delta,$$

and therefore, from eq. (22), the final result immediately follows

$$r_{ij}^{\mathrm{NE,R}} = r_{ij}^{\mathrm{MB,R}}\left[1+\frac{15}{4}\delta-\frac{7}{3}\delta\frac{E_0}{\mathrm{k_B}T}-\left(\frac{E_0}{\mathrm{k_B}T}\right)^2\delta+ +\frac{8}{3}\delta\frac{(E_0-E_\mathrm{R})E_0}{(E_0-E_\mathrm{R})^2+\Gamma_\mathrm{T}^2/4}\frac{E_0}{\mathrm{k_B}T}\right]. \quad (23)$$

4 Conclusions and discussion

In this work we have analytically derived two first-order formulae that can be used to express the non-extensive reaction rate as a product of the classical reaction rate times a suitable corrective factor for both narrow and wide resonances, as shown in eqs. (11) and (23). It should be stressed that the previous results are correct only if $|\delta|$ is very small (in the sense of eq. (16)), namely if we are dealing with slight deformations of the energy distribution function. This is really the most common situation, as far as astrophysical plasmas are concerned (in fact $|\delta| \sim 10^{-3}$–10^{-2}).

Concerning the fusion reactions between two medium-weighted nuclei, for example the $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ reaction, our non-extensive factor, which now can be formally defined as follows.

$$f_\mathrm{NE} = 1+\frac{15}{4}\delta-\left(\frac{E_\mathrm{R}}{\mathrm{k_B}T}\right)^2\delta\,,$$

gives rise to a further correction beside the screening and the potential resonant screening, already investigated in [1] and [3].

It is important to point out that the non-extensivity does not affect the other plasma corrections: therefore, we can define an effective factor F as

$$F = f_\mathrm{NE}\cdot f_\mathrm{S}\cdot f_\mathrm{RS}\,, \quad (24)$$

where f_S and f_RS account for the Debye-Hückel screening and the resonant screening effect, respectively.

We have applied our results to a physical model describing a carbon white-dwarf's plasma, with a temperature of $T = 8\cdot10^8\,\mathrm{K}$ and a mass density of $\rho = 2\cdot10^9\,\mathrm{g\cdot cm^{-3}}$ (the plasma parameter is, correspondingly, $\Gamma \simeq 5.6$). Furthermore, we have set a deformation parameter $|\delta| = 10^{-3}$, regardless of its sign, and we have kept the energy of the possible resonance, E_R, as a free parameter.

In fig. 1, we plot the total corrective factor defined in eq. (24) against the energy at which the narrow resonance is supposed to be located, in the range between 0 keV and 2400 keV, considering three very different conditions (depending on the sign and value of the deformation parameter δ). In our calculations, in order to estimate the functional dependence of $f_\mathrm{S}\cdot f_\mathrm{RS}$ with respect to E_R, we have adopted the fitting formulae derived by Itoh and collaborators [3]: their result, that has been derived through a classical treatment, is also shown in fig. 1 by the line labelled with $\delta = 0$. From the same figure, it is clear that

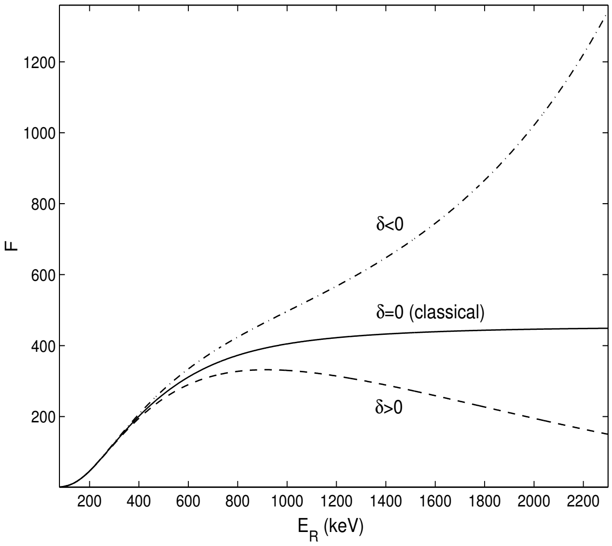

Fig. 1. Linear plot of the effective factor F, defined in eq. (24), against the resonance energy E_R. The dash-dotted (upper) line refers to super-extensivity, the dashed (lower) line to sub-extensivity, while the solid (middle) line describes the classical (MB) result.

a slight non-extensivity does introduce non-trivial corrections that become more and more important as the resonance energy rises: if $\delta > 0$, our total effective factor is $F \simeq 136$ at $E_\mathrm{R} = 2.4\,\mathrm{MeV}$, while, at the same resonance energy, the factor is $F \simeq 1484$ if $\delta < 0$. Anyway, in both cases, $\lim F = 1$ when $E_\mathrm{R} \to 0\,\mathrm{keV}$. It is also noteworthy that the effective factor always acts to enhance the resonant reaction rate, no matter if the system is super- or sub-extensive. In conclusion, all the plasma enhancements due to the presence of long-range many-body nuclear correlations and memory effects, that can be described within the non-extensive statistics by means of the entropic parameter $q > 1$ ($\delta < 0$), are in the direction of still more increasing the effective factor F of nuclear rates of hydrostatic burning and white-dwarfs environment.

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